Streamflow forecasting by SVM with quantum behaved particle swarm optimization

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A B S T R A C T

Accurate forecasting of streamflows has been one of the most important issues as it plays a key role in allotment of water resources. However, the information of streamflow presents a challenging situation; the streamflow forecasting involves a rather complex nonlinear data pattern. In the recent years, the support vector machine has been used widely to solve nonlinear regression and time series problems. This study investigates the accuracy of the hybrid SVM-QPSO model (support vector machine-quantum behaved particle swarm optimization) in predicting monthly streamflows. The proposed SVM-QPSO model is employed in forecasting the streamflow values of Vijayawada station and Polavaram station of Andhra Pradesh in India. The SVM model with various input structures is constructed and the best structure is determined using normalized mean square error (NMSE) and correlation coefficient (R). Further quantum behaved particle swarm optimization function is adapted in this study to determine the optimal values of SVM parameters by minimizing NMSE. Later, the performance of the SVM-QPSO model is compared thoroughly with the popular forecasting models. The results indicate that SVM-QPSO is a far better technique for predicting monthly streamflows as it provides a high degree of accuracy and reliability.

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1. Introduction

Many activities associated with planning and operation of the water resources system depend on the accurate and reliable streamflow forecasts. Streamflow forecasting can be done both in short term like hourly, daily and also in long term like monthly, yearly. From the perspective of the hydrologists both short term forecasts and long term forecasts are significant. In general the storage yield sequences are related to the monthly periods. Therefore, forecasting the monthly flows have greater importance in the water resource system planning. Since the last decade researchers are using the soft computing tools for forecasting complex nonlinear processes. Support vector machine (SVM) is also one such computational technique which has been successfully employed in different areas [1–6]. Likewise even in the field of hydrology researchers have applied SVM extensively [7–15].

This study is motivated by a growing popularity of support vector machine (SVM). Many SVM application studies are performed by ‘expert’ users. Since the quality of SVM models depends on a proper setting of SVM hyper-parameters, the main issue for practitioners trying to apply SVM regression is how to set these parameter values (to ensure good generalization performance) for a given data set. Whereas existing sources on SVM regression [16–18] give some recommendations on appropriate setting of SVM parameters, there is no general consensus and many contradictory opinions. Hence, re-sampling remains the method of choice for many applications. Unfortunately, using re-sampling for tuning several SVM parameters is very expensive in terms of computational costs and data requirements. So there comes a need for an alternate way of searching out the SVM parameters.

PSO was introduced by Eberhart and Kennedy [19], inspired by the social behavior of animals such as bird flocking, fish schooling, and the swarm theory. Compared with GA and other similar evolutionary techniques, PSO has some attractive characteristics and in many cases proved to be more effective [20]. Both GA and PSO have been used extensively for a variety of optimization problems and in most of these cases PSO has proven to have superior computational efficiency [20]. Since 1995, many attempts have been made to improve the performance of the PSO [21,22], Sun and Xu [23] introduced quantum theory into PSO and propose a quantum-behaved PSO (QPSO) algorithm, which is guaranteed theoretically to find good optimal solutions in search space. The experiment results on some widely used benchmark functions show that the QPSO works better than standard PSO [23] and is a promising algorithm. In this study quantum-behaved particle swarm optimization (QPSO) technique is employed to find out SVM parameters. This paper is organized as follows.
Section 2 gives a brief introduction to SVM algorithm. Section 3 describes the adaptation of QPSO algorithm in estimating the optimal SVM parameters. In Section 4 the proposed model is applied over two stations and the results are compared with the popular existing models. Finally, the conclusions are presented in Section 5.

2. Support vector machines

Support vector machine (SVM) is a new and promising technique for data classification and regression. In this section a brief description of SVM is given. Assume \((x_i, y_i) \ldots (x_l, y_l)\) be the given training data sets, where each \(x_i \in R^n\) shows the input space of the sample and has a corresponding target value \(y_i \in R\) for \(i = 1, \ldots , l\) where \(l\) represents the size of the training data. The support vector regression solves an optimization problem:

\[
\begin{align*}
\text{minimize} & \quad \frac{1}{2} \|w\|^2 + C \sum_i \left( \zeta_i^+ + \zeta_i^- \right) \\
\text{subject to} & \quad \langle w, x_i \rangle - b - y_i \leq \epsilon + \zeta_i^+ \\
& \quad \langle w, x_i \rangle - b - y_i \geq \epsilon - \zeta_i^- \\
& \quad \zeta_i^+, \zeta_i^- \geq 0 \quad i = 1, \ldots , l
\end{align*}
\]

(1)

where \(x_i\) is mapped to a higher dimensional space by the function \(\phi\), \(\zeta_i^+\) is the upper training error (\(\zeta_i^-\) is the lower) subject to the \(\epsilon\) insensitive tube \(\langle w, x_i \rangle - b \leq \epsilon\). The parameters which control the regression quality are the cost of error \(C\), the width of the tube \(\epsilon\), and the mapping function \(\phi\). The constraints imply that we would like to put most data \(x_i\) in the tube \(\langle w, x_i \rangle - b \leq \epsilon\). If \(x_i\) is not in the tube, there is an error \(\zeta_i^+\) or \(\zeta_i^-\) which we would like to minimize in the objective function. SVM avoids under fitting and over fitting the training data by minimizing the training error term \(\sum \zeta_i^+ + \zeta_i^-\) as well as the regularization term \(\frac{1}{2}\|w\|^2\). For traditional least square regression \(\epsilon\) is always zero and data are not mapped into higher dimensional spaces. Hence, SVM is a more general and flexible treatment on regression problems.

Many works in forecasting have demonstrated the favorable performance of the radial basis function [7,24,25] as kernel function for SVM. Therefore, the radial basis function (RBF), \(\exp(-\|x-x_i\|^2)\) is adopted in this work. The selection of the three parameters \(\gamma\), \(\epsilon\) and \(C\) of SVM model influence the accuracy of forecasting. However, there is no standard method of selection of these parameters. Therefore, particle swarm optimization technique is used in the proposed model to optimize parameter selection.

3. Quantum behaved particle swarm optimization technique in selecting the parameters of SVM

QPSO was inspired by analysis of the convergence of the traditional PSO and quantum system. In the quantum physics, the state of a particle with momentum and energy can be depicted by its wave function \(\psi(x,t)\). According to QPSO theory each particle is in a quantum state and is formulated by its wave function \(\psi(x,t)\) instead of the position and velocity which are in PSO. According to the statistical significance of the wave function, the probability of a particle’s appearing in a certain position can be obtained from the probability density function \(\psi(x,t)^2\). And then the probability distribution function of the particle's position can be calculated through the probability density function. By employing the Monte Carlo method, the particle's position is updated according to the following equation:

\[
X_j^{t+1} = p_j^t \pm \sqrt{L_j^t} \cdot \ln(1/u_j^t)
\]

(2)

where \(u_j^t\) is a random number uniformly distributed in \((0, 1)\); \(p_j^t\) is the local attractor and defined as

\[
p_j^t = \varphi_j^t\cdot P_g^t + (1-\varphi_j^t) \cdot P_p^t
\]

(3)

where \(\varphi_j^t\) is a random number uniformly distributed in \((0, 1)\), \(P_g^t\) is the global best position. In parameter \(L_j^t\) is evaluated by

\[
L_j^t = 2 \cdot \beta \cdot |p_j^t - X_j^t|
\]

(4)

where parameter \(\beta\) is called the contraction–expansion (CE) coefficient, which can be tuned to control the convergence speed of the algorithms. Then we get the position update equation as

\[
X_j^{t+1} = p_j^t \pm \beta \cdot \varphi_j^t \cdot |p_j^t - X_j^t| \cdot \ln(1/u_j^t)
\]

(5)

The PSO algorithm with position update equation (5) is called as quantum delta-potential-well-based PSO (QDPSO) algorithm. Keeping in view the vital position of \(L\) for convergence rate and performance of the algorithm an improvement was proposed to evaluate parameter \(L\). As per this algorithm the mean best position (mbest) is defined as the center of pbest positions of the swarm. That is

\[
m_{best} = (m_{best1}, m_{best2}, \ldots , m_{bestD})
\]

(6)

Hence, the particle's position is updated according to the following equation:

\[
X_j^{t+1} = p_j^t \pm \beta \cdot |m_{best} - X_j^t| \cdot \ln(1/u_j^t)
\]

(8)

The PSO algorithm with Eq. (8) is called as quantum-behaved particle swarm optimization (QPSO). Pseudo code for implementing the QPSO is given below:

**Algorithm 1.**

Initialize the population size, the positions, and the dimensions of the particles

\[
\text{for} \ t = 1 \rightarrow \text{Maximum iteration} \ T \ \text{do}
\]

Compute the mean best position \(m_{best}\)

\[
\beta = (1.0 - 0.5) \ldots (T - t) / T + 0.5
\]

\[
\text{for} \ i = 1 \rightarrow \text{population size} \ M \ \text{do}
\]

\[
\text{if} \ f(x_i) < f(p_i) \ \text{then}
\]

\[
P_i = X_i
\]

\[
\text{end if}
\]

\[
P_j = \min(P_j)
\]

\[
\text{for} \ j = 1 \rightarrow \text{ dimension} \ D \ \text{do}
\]

\[
\varphi = \text{rand}(0,1); u = \text{rand}(0,1);
\]

\[
p_j = \varphi \cdot P_g + (1-\varphi) \cdot P_p
\]

\[
\text{if rand}(0,1) > 0.5 \ \text{then}
\]

\[
X_i = p_j + \beta
\]

\[
\text{else}
\]

\[
X_i = p_j - \beta \cdot (m_{best} - X_i) \cdot \log(1/u)
\]

\[
\text{end if}
\]

\[
\text{end for}
\]

\[
\text{end for}
\]

The most commonly used control strategy of \(\beta\) is to initially setting it to 1.0 and reducing it linearly to 0.5. In this study the normalized mean square error (NMSE) serves as the fitness criterion for identifying the suitable parameters for SVM model.
The NMSE value of each particle is then determined using the fitness function:

\[
NMSE = \frac{1}{n} \sum_{i=1}^{n} \frac{[Q_m(i) - Q_s(i)]^2}{[Q_m(i)]^2}
\]

(9)

where \(Q\) is the streamflow value and the subscripts ‘\(m\)’ and ‘\(s\)’ represent the measured and simulated values, respectively. The average value of associated variable is represented with a ‘tilde’ above it and \(n\) depicts the total number of training records.

4. Case study and simulation results

In this study, the monthly streamflow data of Polavaram Station on Godavari River and Vijayawada Station on Krishna River of India were used. These two stations are situated on the most important rivers of India. The Godavari River is the second largest river in India, with a catchment area of 312,812 km\(^2\) and a long-term average annual surface flow of 110 km\(^3\). Whereas the Krishna River Basin is the fourth largest in India with a total catchment area of 258,948 km\(^2\) and a long-term average annual surface flow of 78 km\(^3\). Fig. 1 depicts the location of Vijayawada station and Polavaram station. The observed data is 60 years (720 months) long with an observation period between 1901 and 1961 for both stations. Fivefold cross validation test is conducted in this study.

4.1. SVM model development

The goal of SVM model is to generalize a relationship of the form

\[
Z^m = f(X^m)
\]

(10)

where \(X^m\) is an \(n\)-dimensional input vector comprising variables \(x_1, x_2, x_3, \ldots, x_n\) and \(Z^m\) is an \(m\)-dimensional output vector consisting of the resulting variables of interest \(y_1, y_2, y_3, \ldots, y_m\). In modeling the streamflows, the values of \(x_i\) may be streamflow values with different lags and the value of \(y_i\) is the streamflow level of the next time step. However, the number of antecedent streamflow values to be included in the vector \(X^m\) are not known a priori.

Thus in this study, an appropriate input data set is identified by carefully analyzing the various combinations of the streamflow values \(Q\) at various time lags. The input vector is modified each time by successively adding an streamflow value at one more time lag leading to the development of a new SVM model. The appropriate input vector is identified by comparing the coefficient of correlation, efficiency and normalized mean squared error (NMSE). Five SVM models were developed with different sets of inputs variables as follows.

Model 1 \(Q(t) = f[Q(t-1)]\)
Model 2 \(Q(t) = f[Q(t-1), Q(t-2)]\)
Model 3 \(Q(t) = f[Q(t-1), Q(t-2), Q(t-3)]\)
Model 4 \(Q(t) = f[Q(t-1), Q(t-2), Q(t-3), Q(t-4)]\)
Model 5 \(Q(t) = f[Q(t-1), Q(t-2), Q(t-3), Q(t-4), Q(t-5)]\)

4.1.1. Determining parameters of SVM models

The methodology of estimating SVM parameters with the help of QPSO is mentioned in Section 3. Due to a lack of any priori knowledge on the bounds of SVM parameters, a two-step QPSO search algorithm is adapted [26]. In the first step, a coarse range search is made to achieve the best region of the three-dimensional grids. Since doing a complete grid-search may still be time-consuming, a coarse grid search is recommended first. After identifying a better region on the grid, a finer grid search on that region can be conducted. In the present study, the coarse range partitions for \(C\) are taken as \([10^{-5}, 10^5]\) [25]. Similarly, the coarse range partitions for \(\epsilon\) are taken to be \([0, 10^4]\) and the coarse range partitions for \(\gamma\) are \([0, 10^4]\) [26,25]. Once the better region of grid is determined then a search is conducted in the finer range.

### Table 1
Optimal SVM parameters obtained from QPSO for different models for Vijayawada station.

<table>
<thead>
<tr>
<th>Model</th>
<th>(C)</th>
<th>(\epsilon)</th>
<th>(\gamma)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td>1.62</td>
<td>0.121</td>
<td>0.975</td>
</tr>
<tr>
<td>Model 2</td>
<td>1.056</td>
<td>0.017</td>
<td>0.6</td>
</tr>
<tr>
<td>Model 3</td>
<td>1.76</td>
<td>0.065</td>
<td>0.355</td>
</tr>
<tr>
<td>Model 4</td>
<td>1.53</td>
<td>0.154</td>
<td>0.29</td>
</tr>
<tr>
<td>Model 5</td>
<td>1.82</td>
<td>0.089</td>
<td>0.83</td>
</tr>
</tbody>
</table>

### Table 2
Optimal SVM parameters obtained from QPSO for different models for Polavaram station.

<table>
<thead>
<tr>
<th>Model</th>
<th>(C)</th>
<th>(\epsilon)</th>
<th>(\gamma)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td>1.19</td>
<td>0.678</td>
<td>0.24</td>
</tr>
<tr>
<td>Model 2</td>
<td>1.6</td>
<td>0.43</td>
<td>0.87</td>
</tr>
<tr>
<td>Model 3</td>
<td>1.48</td>
<td>0.36</td>
<td>0.41</td>
</tr>
<tr>
<td>Model 4</td>
<td>1.64</td>
<td>0.415</td>
<td>0.78</td>
</tr>
<tr>
<td>Model 5</td>
<td>1.91</td>
<td>0.095</td>
<td>0.94</td>
</tr>
</tbody>
</table>

### Table 3
The average NMSE and R statistics of SVM-QPSO application for Vijayawada station.

<table>
<thead>
<tr>
<th>Model inputs</th>
<th>NMSE</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Q_{t-1}, Q_{t-2})</td>
<td>0.159</td>
<td>0.92</td>
</tr>
<tr>
<td>(Q_{t-1}, Q_{t-2}, Q_{t-3})</td>
<td>0.204</td>
<td>0.918</td>
</tr>
<tr>
<td>(Q_{t-1}, Q_{t-2}, Q_{t-3}, Q_{t-4})</td>
<td>0.1251</td>
<td>0.945</td>
</tr>
<tr>
<td>(Q_{t-1}, Q_{t-2}, Q_{t-3}, Q_{t-4}, Q_{t-5})</td>
<td>0.13</td>
<td>0.93</td>
</tr>
<tr>
<td>(Q_{t-1}, Q_{t-2}, Q_{t-3}, Q_{t-4}, Q_{t-5})</td>
<td>0.148</td>
<td>0.929</td>
</tr>
</tbody>
</table>
Thus in the second stage search the parameter $C$ ranges between $[10^{-3}, 10^3]$; $c$ is taken to be $[10^{-2}, 10^{-1}]$ and $\gamma$ is taken to be $[0,1]$.

For the Vijayawada station, five input combinations based on preceding monthly streamflows are evaluated to estimate current streamflow value. PSO is used to find parameters for all the five cases. The optimal set of SVM parameters obtained from PSO algorithm for Vijayawada station is given in Table 1. The average NMSE and $R$ statistics of SVM-PSO models obtained after the 5 fivefold cross validation test are given in Table 3. The table indicates that the SVM-PSO model whose inputs are the flows of three previous months ($Q(t) = f(Q(t-1), Q(t-2), Q(t-3))$) has the best accuracy compared to the other models. Also it is seen that adding fourth and fifth lags is not improving the model prediction.

Similarly for the Polavaram station also five SVM-PSO models are developed to estimate current streamflow value. Table 2 depicts the parameters of the SVM model obtained from QPSO technique after optimizing Eq. (9). Further NMSE and $R$ statistics are computed which are presented in Table 4. The table indicates that SVM-PSO model with three antecedent streamflows ($Q(t) = f(Q(t-1), Q(t-2), Q(t-3))$) is showing better accuracy when compared with the other models. It is found that adding fourth and fifth lags is not much improving the performance.

### 4.2. Models for comparing forecast performance

The normalized mean squared error (NMSE) as given in Eq. (9) is used as the measurement of forecasting accuracy. Additionally, the accurate efficiency is measured by coefficient of correlation ($R$) given by

$$R = \frac{\sum_{t=1}^{n}(Q_m - \bar{Q}_m)(Q_s - \bar{Q}_s)}{\sqrt{\sum_{t=1}^{n}(Q_m - \bar{Q}_m)^2 \sum_{t=1}^{n}(Q_s - \bar{Q}_s)^2}}$$

where $Q$ is the streamflow value and the subscripts 'm' and 's' represent the measured and simulated values, respectively. The average value of associated variable is represented with a 'tilde' above it and $n$ depicts the total number of training records.

The forecasting accuracy of the proposed SVM-QPSO model is compared with auto regressive moving average model (ARMA), neural networks (ANN) and SVM-PSO. The performances of SVM-QPSO, SVM-PSO and ANN models are compared while predicting the streamflow values taking a one month lead. Similar to the SVM-QPSO model, PSO also has been used to estimate the parameters of SVM. For PSO and QPSO the population size of 20–40 is considered [23,19]. Further time varying weights and acceleration factors have been implemented in this work [19]. In general neural networks suffer from the problem of potential convergence to a local minimum and are thus susceptible to easy over fitting. Thus in this study the fivefold cross validation is adapted to avoid over fitting and then the optimum architectures for ANN have been determined. The architectures for the ANN model for Vijayawada station is 3–20–1 and for Polavaram station

### Table 4

<table>
<thead>
<tr>
<th>Model inputs</th>
<th>NMSE</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_{t-1}$</td>
<td>0.2195</td>
<td>0.868</td>
</tr>
<tr>
<td>$Q_{t-1}$, $Q_{t-2}$</td>
<td>0.2177</td>
<td>0.888</td>
</tr>
<tr>
<td>$Q_{t-1}$, $Q_{t-2}$, $Q_{t-3}$</td>
<td>0.1</td>
<td>0.951</td>
</tr>
<tr>
<td>$Q_{t-1}$, $Q_{t-2}$, $Q_{t-3}$, $Q_{t-4}$</td>
<td>0.19</td>
<td>0.94</td>
</tr>
<tr>
<td>$Q_{t-1}$, $Q_{t-2}$, $Q_{t-3}$, $Q_{t-4}$, $Q_{t-5}$</td>
<td>0.205</td>
<td>0.93</td>
</tr>
</tbody>
</table>

### Table 5

<table>
<thead>
<tr>
<th>Forecasting Models</th>
<th>Vijayawada station</th>
<th>Polavaram station</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$R$</td>
<td>NMSE</td>
</tr>
<tr>
<td>SVM-QPSO</td>
<td>0.944</td>
<td>0.12</td>
</tr>
<tr>
<td>SVM-PSO</td>
<td>0.92</td>
<td>0.19</td>
</tr>
<tr>
<td>SVM</td>
<td>0.823</td>
<td>0.3965</td>
</tr>
<tr>
<td>ANN</td>
<td>0.88</td>
<td>0.41</td>
</tr>
<tr>
<td>ARMA</td>
<td>0.83</td>
<td>0.3189</td>
</tr>
</tbody>
</table>

### Table 6

<table>
<thead>
<tr>
<th>Peak No.</th>
<th>Observed streamflow (m$^3$/s)</th>
<th>ARMA</th>
<th>ANN</th>
<th>SVM</th>
<th>SVM-QPSO</th>
<th>Relative error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>ARMA</td>
</tr>
<tr>
<td>1</td>
<td>12,247</td>
<td>4147.0</td>
<td>5144.8</td>
<td>6026.0</td>
<td>4572.6</td>
<td>7053.6</td>
</tr>
<tr>
<td>2</td>
<td>11,988</td>
<td>4137.2</td>
<td>5199.6</td>
<td>4144.0</td>
<td>5571.9</td>
<td>7775.5</td>
</tr>
<tr>
<td>3</td>
<td>11,891</td>
<td>4166.5</td>
<td>5379.5</td>
<td>4963.0</td>
<td>7146.0</td>
<td>5987.2</td>
</tr>
<tr>
<td>4</td>
<td>11,375</td>
<td>6191.0</td>
<td>5000.0</td>
<td>6118.0</td>
<td>7229.4</td>
<td>9190.3</td>
</tr>
<tr>
<td>5</td>
<td>9217</td>
<td>4384.5</td>
<td>7252.7</td>
<td>4822.0</td>
<td>4892.3</td>
<td>5842.1</td>
</tr>
<tr>
<td>6</td>
<td>8383</td>
<td>6181.0</td>
<td>5895.9</td>
<td>8414.0</td>
<td>6052.6</td>
<td>8491.4</td>
</tr>
<tr>
<td>7</td>
<td>8225</td>
<td>6151.7</td>
<td>6840.5</td>
<td>6302.0</td>
<td>7962.6</td>
<td>10478</td>
</tr>
<tr>
<td>8</td>
<td>8822</td>
<td>6200.5</td>
<td>4183.2</td>
<td>5541.0</td>
<td>7017.8</td>
<td>8540.5</td>
</tr>
<tr>
<td>9</td>
<td>7176</td>
<td>6210.3</td>
<td>5897.0</td>
<td>5776.0</td>
<td>6355.4</td>
<td>7544.2</td>
</tr>
<tr>
<td>10</td>
<td>6931</td>
<td>6161.5</td>
<td>6711.2</td>
<td>4192.0</td>
<td>7052.0</td>
<td>7188.1</td>
</tr>
<tr>
<td>11</td>
<td>6870</td>
<td>4394.2</td>
<td>4052.6</td>
<td>7660.0</td>
<td>4637.8</td>
<td>6346.4</td>
</tr>
</tbody>
</table>

Mean relative absolute peak prediction error

0.375 | 0.362 | 0.344 | 0.29 | 0.214
it is 3–15–1. ARIMA model has been developed using a open source software ITSM and therefore has the best possible combination of parameters.

For Vijayawada station comparison of the performance of the single month lead forecasting resulting from ARMA, ANN, SVM, SVM-PSO and SVM-QPSO methods is depicted in Table 5.

Table 5 is comparing the performance values for different forecasting models with the proposed SVM-QPSO model. SVM-QPSO has a NMSE of 0.12 whereas it is 0.199 for SVM-PSO, 0.39 for SVM, 0.41 for ANN, and 0.318 for ARMA process. Thus SVM-QPSO shows an improved performance compared to other models. Further it is seen that SVM if the parameters are not properly determined then its performance is poor even compared to ANN, as shown in Table 5. Similarly the correlation coefficient also improves when one moves from ARMA, ANN, SVM, SVM-PSO to SVM-QPSO. As SVM-QPSO predicts the streamflow values with a R value of 0.944 while SVM-PSO model exhibits R value of 0.92, ANN has R value of 0.88 and for ARMA model it is 0.83. Further Fig. 2 shows the comparison of the measured and predicted heads using the testing data for SVM-PSO and SVM-QPSO methods. Furthermore in this study we have calculated the mean absolute peak prediction error. Table 6 depicts the error involved in predicting the peak streamflows by the each process. It is seen from Table 6 that although all the models are underestimating the peaks but SVM-QPSO is predicting peak with minimum error of 0.214 where as SVM-PSO has 0.491, SVM is having 0.56, ARMA is having 0.561 and ANN is having 0.543 error in predicting peaks.

Similar analysis has been conducted for Polavaram station. Table 5 is showing the performance statistics of all the models. SVM-QPSO has NMSE of 0.1, where as SVM-PSO has NMSE of 0.199, ANN is predicting streamflow values with NMSE of 0.338 and further ARMA has NMSE of 0.5. This shows that SVM-QPSO is predicting the streamflow values with less error. Further the R value of SVM-QPSO is close to 1 compared to SVM-PSO, SVM, ARMA and ANN. Also Fig. 3 shows the comparison of the measured and predicted heads using the testing data for SVM-PSO and SVM-QPSO methods. Besides this Mean absolute peak prediction error has been calculated to determine which method is predicting peaks more closely. From Table 7 it is observed that SVM-QPSO predicts peaks with 0.188, while SVM-PSO has 0.27 error and SVM has 0.321 error. ARMA model predicts peaks with 0.48 error and ANN has 0.37 error. Again it is showing that improper selection of SVM parameters lead to a poor performance. Therefore, the non-linear mapping ability and proper selection of parameters make the SVM-QPSO successful in streamflow forecasting.

The performance of SVM-QPSO is out performing compared to other models. This is because of unique combination of SVM and QPSO. Further, the introduction of mean best position into QPSO is an innovative method. In Original PSO, each particle converges to the global best position independently. On the other hand, in the QPSO with mean best position mbest, each particle cannot converge to the global best position without considering its colleagues for there are some waiting among the particles. It is because that the distance between particle’s current position and mbest determines the position distribution of the particle for next iteration. If the personal best positions of several particles are far from the global best position (these particles called lagged particles) then the position of mbest may be pulled away from the global best position by lagged particles. When the lagged particles are chasing after their colleagues, say converging to the global best position, the position mbest will be approaching the global best position slowly. The distances between position mbest and the personal best positions of a particle near the global best position do not decrease quickly, decelerating the convergence of the particles near the global best position and making them explore globally around the global best position temporarily until the lagged ones are close to the global best position. Therefore, in the QPSO with mean best position, the particle swarm never abandons any lagged particle and seems to be more intelligent and more cooperative social organism. In a word, the wait mechanism

![Fig. 3. Comparison between simulated and actual streamflow values of Polavaram station.](image)

<table>
<thead>
<tr>
<th>Peak No.</th>
<th>Observed streamflow (m$^3$/s)</th>
<th>ARMA</th>
<th>ANN</th>
<th>SVM</th>
<th>SVM-PSO</th>
<th>SVM-QPSO</th>
<th>Relative error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>32,733</td>
<td>6710</td>
<td>13,922</td>
<td>10,715</td>
<td>14,455</td>
<td>20,838</td>
<td>0.795, 0.574, 0.673, 0.558, 0.36</td>
</tr>
<tr>
<td>2</td>
<td>26,479</td>
<td>8010</td>
<td>13,367</td>
<td>11,857</td>
<td>18,344</td>
<td>28,715</td>
<td>0.697, 0.495, 0.552, 0.307, 0.08</td>
</tr>
<tr>
<td>3</td>
<td>20,188</td>
<td>8330</td>
<td>8640</td>
<td>11,453</td>
<td>12,903</td>
<td>20,575</td>
<td>0.587, 0.571, 0.433, 0.36, 0.02</td>
</tr>
<tr>
<td>4</td>
<td>18,329</td>
<td>6840</td>
<td>5274</td>
<td>9795</td>
<td>12,303</td>
<td>13,667</td>
<td>0.626, 0.712, 0.466, 0.328, 0.25</td>
</tr>
<tr>
<td>5</td>
<td>17,640</td>
<td>8500</td>
<td>14,446</td>
<td>12,551</td>
<td>12,126</td>
<td>14,257</td>
<td>0.518, 0.181, 0.288, 0.312, 0.19</td>
</tr>
<tr>
<td>6</td>
<td>17,507</td>
<td>7260</td>
<td>12,604</td>
<td>10,714</td>
<td>15,654</td>
<td>12,232</td>
<td>0.585, 0.28, 0.388, 0.105, 0.3</td>
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<tr>
<td>7</td>
<td>14,001</td>
<td>6140</td>
<td>11,842</td>
<td>9277</td>
<td>11,306</td>
<td>9118</td>
<td>0.561, 0.154, 0.337, 0.192, 0.35</td>
</tr>
<tr>
<td>8</td>
<td>13,980</td>
<td>8670</td>
<td>18,818</td>
<td>11,644</td>
<td>15,403</td>
<td>14,133</td>
<td>0.379, 0.346, 0.167, 0.101, 0.01</td>
</tr>
<tr>
<td>9</td>
<td>11,324</td>
<td>7860</td>
<td>11,212</td>
<td>11,807</td>
<td>9811</td>
<td>10,869</td>
<td>0.305, 0.009, 0.043, 0.133, 0.04</td>
</tr>
<tr>
<td>10</td>
<td>10,722</td>
<td>7400</td>
<td>13,559</td>
<td>9488</td>
<td>11,916</td>
<td>9541</td>
<td>0.309, 0.264, 0.115, 0.111, 0.11</td>
</tr>
<tr>
<td>11</td>
<td>10,668</td>
<td>5790</td>
<td>19,234</td>
<td>8052</td>
<td>12,672</td>
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<td>0.457, 0.802, 0.245, 0.187, 0.62</td>
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<tr>
<td>12</td>
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<td>8840</td>
<td>14,438</td>
<td>12,553</td>
<td>12,332</td>
<td>10,191</td>
<td>0.156, 0.377, 0.197, 0.176, 0.03</td>
</tr>
<tr>
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<td>10,397</td>
<td>2650</td>
<td>7733</td>
<td>4233</td>
<td>6708</td>
<td>9222</td>
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<tr>
<td>14</td>
<td>10,193</td>
<td>8170</td>
<td>12,434</td>
<td>11,913</td>
<td>16,551</td>
<td>12,018</td>
<td>0.198, 0.219, 0.169, 0.623, 0.18</td>
</tr>
<tr>
<td>15</td>
<td>9693</td>
<td>5900</td>
<td>6630</td>
<td>8106</td>
<td>7771</td>
<td>8146</td>
<td>0.391, 0.316, 0.164, 0.198, 0.16</td>
</tr>
</tbody>
</table>

Mean relative absolute peak prediction error: 0.48, 0.37, 0.321, 0.27, 0.188.
among particles enhances the global search ability of QPSO and thus the accuracy of SVM-QPSO models are more accurate compared to the other models.

5. Conclusions

The accuracy of SVM-QPSO model has been investigated for forecasting monthly streamflows in the present study. The SVM-QPSO models were obtained by combining two methods QPSO and SVM. SVM conducts structural minimization rather than the minimization of the errors. The QPSO selects the optimal parameters for SVM to improve the forecasting accuracy. So this unique combination of SVM and QPSO has made the proposed SVM-QPSO model to perform better compared to the other traditional models. The performance of SVM-QPSO is out performing compared to other models. In addition to this SVM-QPSO model exhibits better performance than SVM-PFO. This because the modified version of PSO is much efficient compared to the simple PSO algorithm. Also it is seen from the results that SVM performs poorly in case the parameters are not chosen properly. Furthermore this investigation demonstrates that the proposed SVM-QPSO offers a valid alternative for application in hydrology. In this study univariate time series analysis has been performed. In future a multivariate time series analysis can be done considering the effect of other hydrological variables.

References


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